Shafts

4.1 Power transmission shafting

Continuous mechanical power is usually transmitted along and between rotating shafts. The transfer between shafts is accomplished by gears, belts, chains or other similar means for matching the torque/speed characteristics of the interconnected shafts, e.g. a car needs gears between the engine crankshaft and drive wheel half-shafts. Shafts rotating only at constant speed N (rev/s) are considered here.

Power = force (N) × linear velocity (m/s) in translational applications and
Power = torque (Nm) × angular velocity (= 2π N rad/s) in rotational applications, then it follows that torque is a major load component in power transmitting rotating shafts.

4.2 Torque transmission

Torque may be transferred to or from the end of one shaft by a second coaxial shaft - this is a pure torque, a twist about the shaft axis. The transfer is carried out by a shaft coupling , see fig.(4.1).

Torque may be transferred also at any point along a shaft by a gear, belt pulley, or chain sprocket for example, mounted on the shaft. These common elements apply forces offset from the shaft axis, and therefore the torque (T) is accompanied by a radial load which results in bending

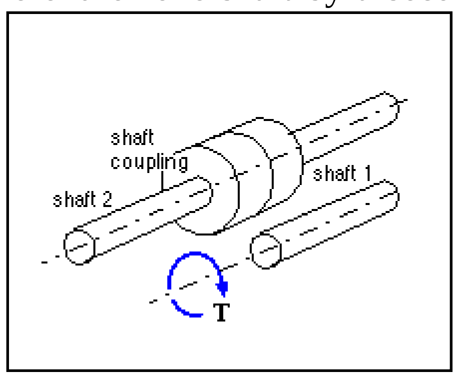

Fig.(4.1)

A spur gear and a belt pulley are sketched in fig.(4.2), each subjected to loading tangential to its ***effective*** or ***pitch*** diameter D. The load on the spur gear arises from inter-tooth contact with its mating gear and comprises two components, the useful tangential component F_t and the unwanted but unavoidable radial component F_r (commonly 0.36 F_t). Gear forms other than spur give rise also to a load component parallel to the shaft axis - but for all gears, shifting the offset force as above, T = $F_t D/2$. See fig. (4.2)

A belt, being flexible, cannot withstand compression - the pulley is therefore subjected to two strand tensions F_{max} and F_{min} both of which must exceed zero. The net torque $T = (F_{max} - F_{min}) D/2$ is clockwise here. A chain sprocket is similar though the minimum tension may drop to zero due to the positive drive not relying on friction.

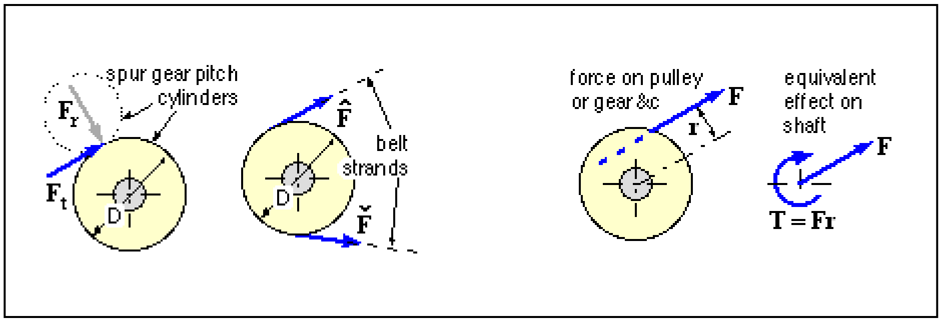

Fig. (4.2)

4.3 Design of shafts under various types of loading

The following equations can be used to obtain the size of the shaft under various types of loadings

Shafts under torsion only:

$$D = B_3\sqrt[3]{\frac{5.1K_tT}{S_s}} \qquad (4.1)$$

When using power

$$D = B_3\sqrt[3]{\frac{321000K_tP}{S_sN}} \qquad (4.1b)$$

Shafts under pure bending

$$D = B_3\sqrt[3]{\frac{10.2K_mM}{S}} \qquad (4.2)$$

Shafts under bending and torsion:

$$D = B_3\sqrt[3]{\frac{5.1}{p_t}\sqrt{(K_mM)^2+(K_tT)^2}} \qquad (4.3a)$$

When using power

$$D = B_3\sqrt[3]{\frac{5.1}{p_t}\sqrt{(K_mM)^2+\left(\frac{63000K_tP}{N}\right)^2}} \qquad (4.3b)$$

Short shafts under transverse shear only

$$D = \sqrt{\frac{1.7V}{S_s}} \qquad (4.4)$$

When using metric system then equation (4.1b) becomes

$$D = B_3\sqrt[3]{\frac{48.7K_tP}{S_sN}} \qquad (4.5)$$

And equation (4.3b) becomes

$$D = B\sqrt[3]{\frac{5.1}{p_t}\sqrt{(K_m M)^2 + \left(\frac{9.55 K_t P}{N}\right)^2}} \qquad (4.6)$$

Where,

D = external diameter of shaft in inch

D_1= internal diameter of shaft in inch

K = D_1/D (for hollow shafts)

$B = \sqrt[3]{1 \div (1 - K^4)}$

K_m = combined factor of shock and fatigue under bending

K_t = combined factor of shock and fatigue under torsion

M = maximum bending moment (in lb)

T = maximum torque (in lb)

N = rotational speed (rpm)

P = Power (hp)

p_t = maximum allowable shear stress under combined load of bending and torsion (psi)

S =maximum allowable bending stress (psi)

S_s =maximum allowable shear stress (psi)

V =maximum allowable transverse shear stress (psi)

Table (4.1) Combined Shock and Fatigue Factors for Various Types of Load

Type of load	Stationary shafts		Rotating shafts	
	K_m	K_t	K_m	K_t
Constant loads without shocks	1.0	1.0	1.5	1.0
Sudden loads with light shocks	1.5 -- 2.0	1.5 -- 2.0	1.5 -- 2.0	1 -- 1.5
Sudden loads with heavy shocks	……	……	2.0 -- 3.0	1.5 -- 3.0

Table (4.2) Recommended Maximum Allowable Stresses for Shafts Under Various Types of Loads

Material	Type of load		
	Bending only	Torsion only	Torsion + bending
Commercial steel Without key way	S=16000 psi *(110 N/mm²)*	S_s = 8000 psi *(55 N/mm²)*	p_t = 8000 psi *(55 N/mm²)*
Commercial steel With key way	S = 12000 psi *(83 N/mm²)*	S_s = 6000 psi *(41 N/mm²)*	p_t = 6000 psi *(41 N/mm²)*
Steel with specific properties	Note(a)	Note(b)	Note(b)

(a) S= 60% of elastic limit in tension and not more than 36% of ultimate tensile strength

(b) S_s and p_t = 30% of elastic limit in tension and not more than 18% of ultimate tensile strength

Table (4.3) Values of the Factor B Corresponding to Various Values of K for Hollow Shafts

$K = D_1/D$	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.95	0.90
$B = \sqrt[3]{1 \div (1 - K^4)}$	1.02	1.03	1.05	1.07	1.10	1.14	1.19	1.28	1.75	1.43

Example 1

A shaft carrying two pulleys is shown in fig.(4.3).The shafts transmits a torque of 300 in.lb. and carries sudden loads at A and B equivalent to 250 lb and 150 lb. respectively, with minor shocks. The two pulleys are fixed to the shaft through rectangular keys. The hub length of pulley A is 2 inch and that of pulley B is 1 inch. If total length of the shaft is 20 inch and it is made of commercial steel find the proper diameter of the shaft in all sections.

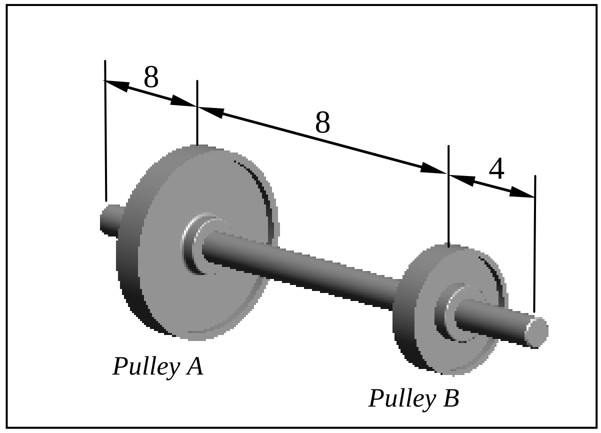

Fig.(4.3)

General guidelines:

1. Make the shaft as short as possible while locating the bearings as close as possible to the loads. This reduces bending moments and deformation and increases critical speed.
2. Eliminate stress raisers near highly stresses areas if possible otherwise use large filets and improve surface finish. Also cold rolling and shot peening can be used to improve the mechanical properties of the material at these areas.
3. If deformation is the main design factor using expensive steel does not solve the problem as all steels have almost the same modulus of elasticity
4. If the weight of the shaft is an important factor the using hollow shafts may give satisfactory solution, eg shaft propeller in cars

Exercises:

1. Figure (4.6) shows the loads acting on a shaft. The shaft caries two pulleys a and b. The loads on shaft are 300 lb at pulley A and 100 lb at pulley B and a maximum torque of 250 in lb. The shaft is made of steel SAE 1010 with Elastic limit of 31000 psi and shear strength of 20000 psi. If the shaft drives a compressor and the 1.5 thick pulley are fixed to the shaft through keys, find the suitable diameters at all sections of the shaft. Take factor of safety = 1.5

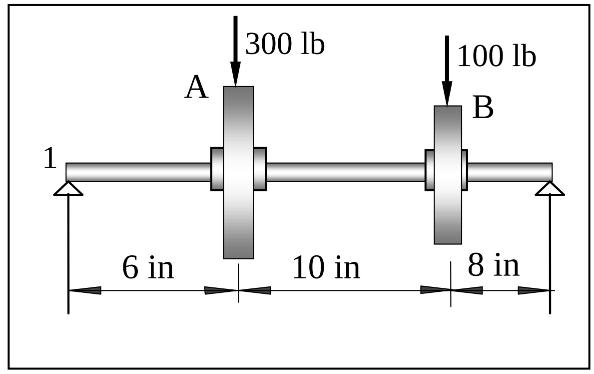

Fig. (4.6)

2. A shaft carries a single gear and transmits a power of 1.5 kW at a speed of 1500 rpm. The gear exerts a tangential load of 500 N and a radial load of 210 N on the shaft. The total length of the shaft is 250 mm and it is carried by two bearings at its ends while the gear is fixed at its centre with a square key. The shaft is connected to a centrifugal pump at one of its ends. Determine suitable diameters for the shaft at its sections

3. Find the diameters of a transmission shaft connected to a six-cylinder 100 hp oil engine through a belt drive, see fig,(4.7). The shaft is driving a woodworking and metalworking machinery and runs at 225 rpm. The torque transmitted through the shaft is divided equally between the working machinery. Take a safety factor of 1.25

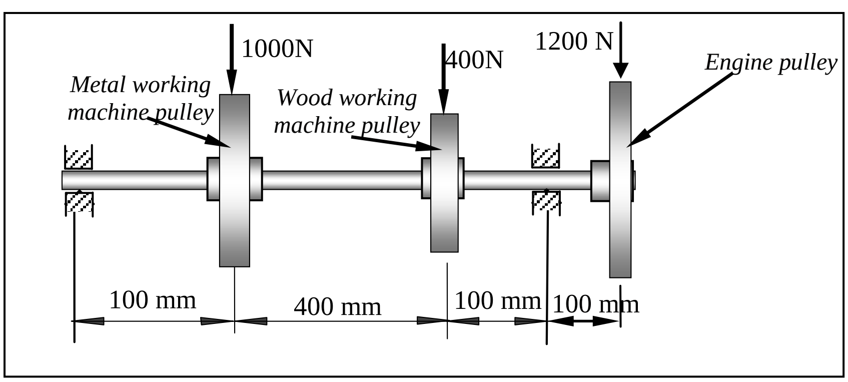

Fig. (4.7)

5. A geared industrial roll shown in fig.(4.8) is driven at 300 rpm by a force F acting on a 3in diameter pitch circle as shown. The roll exerts a normal force of 30 lb/in of roll length on the material being pulled through. The material passes under the roll. The coefficient of friction is 0.4. Develop the moment and shear diagrams for the shaft modelling the roll force as:
a) a concentrated force at the centre of the roll and,
b) a uniformly distributed force along the roll.
Select material and determine dimensions of the shaft if the roll is fitted to the shaft through a square key

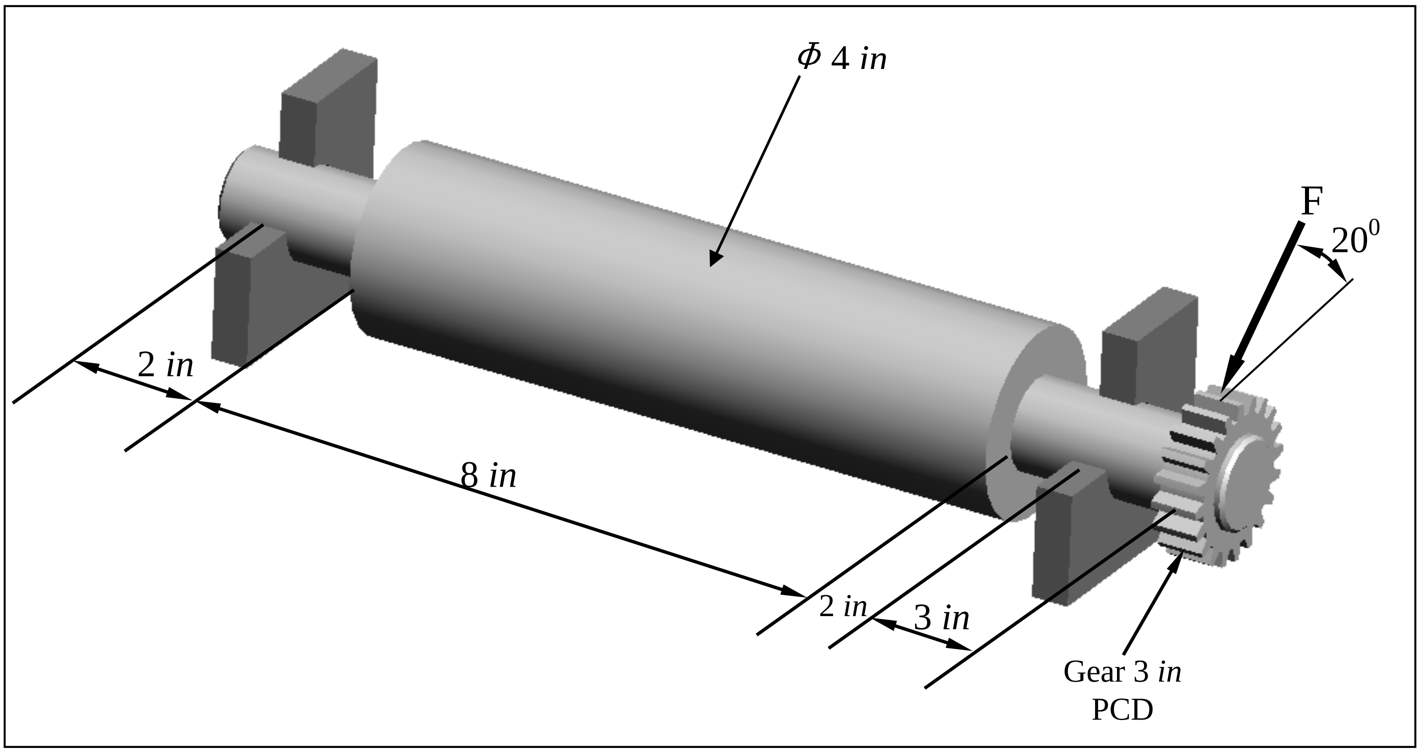

Fig. (4.8)

Keys and pins

5.1 Types of keys

The main function of a key is to transmit torque between a shaft and a machine part assembled to it. In most cases keys prevent relative motion, both rotary and axial. In some construction they allow axial motion between the shaft and the hub; such keys are called feather keys or spline keys.

Keys can be classified according to their shape into straight, and tapered, rectangular, square, round, and dovetail.

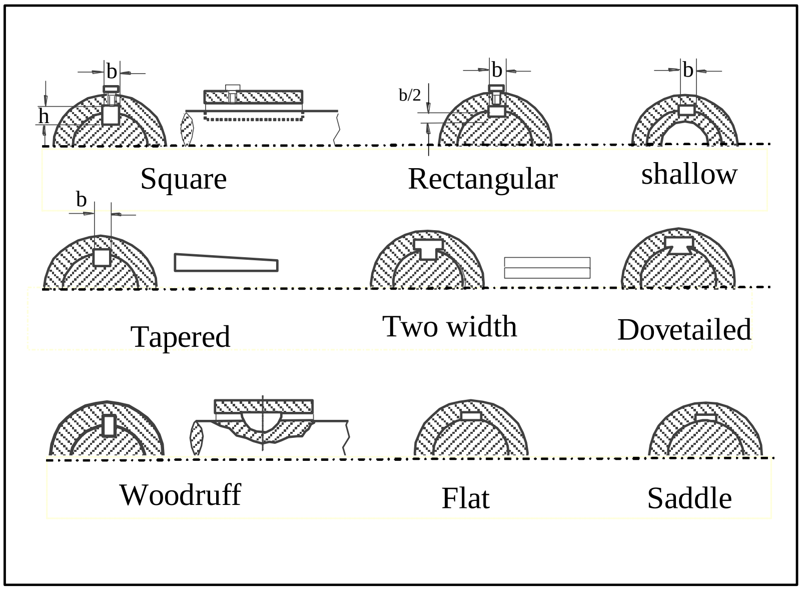

Fig. (5.1) Shaft keys for light and medium duty

Keys are also classified according to their intended duty as:

1. Light duty keys, square: *rectangular key, shallow key*. See fig. (5.1)
2. Medium duty: *taper key.*
3. Heavy duty keys: r*ound tapered key, Barth key*. See fig. (5.2)

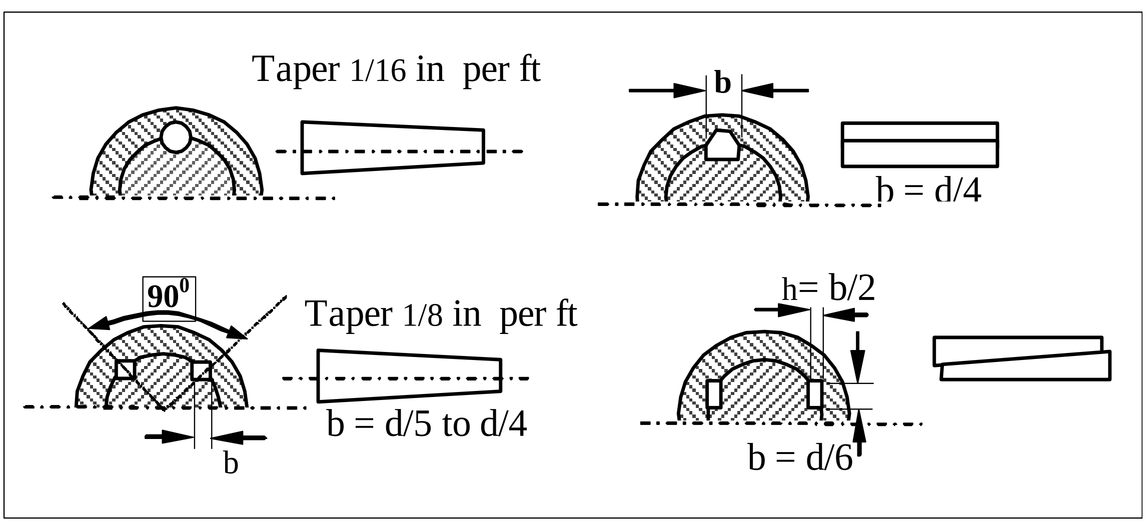

Fig. (5.2) Heavy duty keys

5.2 Design of square keys:

When torque is transmitted through keys, they are subjected to shear and compressive crushing stresses. See fig. (5.3)

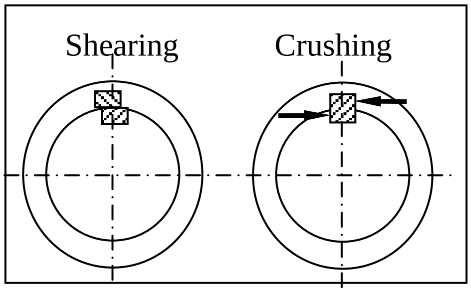

Fig.(5.3)

Crushing strength:

Since a hub is always much more rigid than a shaft, the shaft will be twisted by the torque whereas the hub will remain practically undistorted. As a result the pressure along the key will vary and it will be minimum at the free end of the shaft and maximum on the other side. The maximum pressure can be denoted by P_1 while the minimum pressure P_2 and the pressure at any point along the key by P. So at a distance L_o the pressure equals to zero (see fig (5.4). The pressure can be expressed by the equation:

$$P = P_1 - L \tan\alpha \qquad (5.1)$$

Where

$$\tan\alpha = \frac{(P_1 - P_2)}{L_2} = \frac{P_1}{L_0} \qquad (5.2)$$

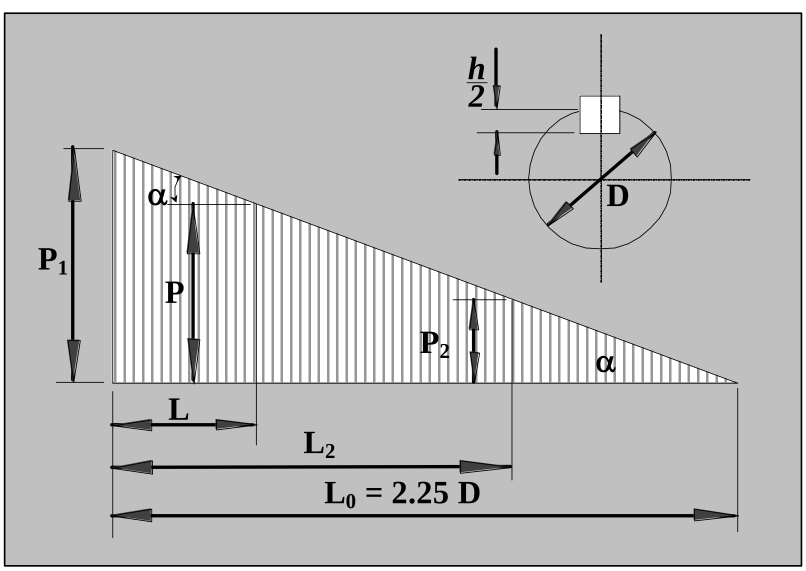

Fig. (5.4)

Torque transmitted

$$dT = PxdLx\ \tfrac{1}{2}\ D \qquad (5.3)$$

Substituting the value of P from equation (5.1) into equation (5.3) and integrating between the limits L = 0 to L = L_2 yields:

$$T = \tfrac{1}{2} P_1 DL_2 - \tfrac{1}{4} DL_2^2 \tan\alpha \qquad (5.4)$$

The pressure /unit length equals the crushing stress x the area of unit length of the key side, (S_bx0.5hx1) then,

$$P_1 = 0.5 S_b h$$

Experiments showed that length of key greater than 2.25D is not effective.
So we can consider that the pressure at L=2.25D equals zero and hence,

$$\tan\alpha = \frac{P_1}{L_0} = \frac{S_b h}{4.5D} \qquad (5.5)$$

And the torque transmitted can be expressed by,

$$T = \tfrac{1}{4} S_b hDL_2 - \tfrac{1}{18} S_b hL_2^2 \qquad (5.6)$$

The length of the key can be determined from equation (5.6). If the outcome is negative value then one key is not enough, but if L< D then take L = D.

Shear strength

The strength of the key can be represented by a diagram similar to fig.(5.4) and with $P_1 = S_s b$ where S_s is the maximum shear at the end of the key and hence

$$\tan\alpha = \frac{P_1}{L_0} = \frac{S_s h}{2.25D} \qquad (5.7)$$

And the torque transmitted can be expressed by:

$$T = \tfrac{1}{2} S_S bDL_2 - \tfrac{1}{9} S_S bL_2^2 \qquad (5.8)$$

From equation (5.8)

$$S_{S=} \frac{T}{L_2 b(0.5D - 0.11L_2)} \qquad (5.9)$$

Based on the diameter of the shaft the standard dimensions of a square can be determined from table (5.1a) or table (5.1b)

The maximum length of the key should not exceed 2.25 D as the extra length, practically, will be useless

Table (5.1a) Standard dimensions of straight key (metric)

Diameter of Shaft inclusive (mm)	Key Dimensions (mm)		Diameter of Shaft inclusive (mm)	Key Dimensions (mm)	
	Width b	Thickness h		Width b	Thickness h
6 - 8	2	2	86 – 95	25	14
9 – 10	3	3	96 – 110	28	16
11 – 12	4	4	111 – 130	32	18
13 – 17	5	5	131 – 150	36	20
18 – 22	6	6	151 – 170	40	22
23 – 30	8	7	171 – 200	45	25
31 – 38	10	8	201 – 230	50	28
39 – 44	12	8	231 – 260	50	32
45 – 50	14	9	261 – 290	63	32
51 – 58	16	10	291 – 330	70	36
59 – 65	18	11	331 – 380	80	40
66 – 75	20	12	381 – 400	90	45
76 - 85	22	14	401 - 500	100	50

Table (5.1b) Standard dimensions of straight keys (inch)

Diameter of shaft D (in) (Inclusive)	Key dimensions (in)			Diameter of shaft D (in) (Inclusive)	Key dimensions (in)		
	Width b	Thickness			Width b	Thickness	
		Standard h	Flat h'			Standard h	Flat h'
$\frac{1}{2}---\frac{9}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{32}$	$3\frac{3}{8}--->3\frac{13}{16}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{5}{8}$
$\frac{5}{8}--->\frac{7}{8}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{8}$	$3\frac{7}{8}--->4\frac{11}{16}$	1	1	$\frac{3}{4}$
$\frac{15}{16}--->1\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{16}$	$4\frac{3}{4}--->5\frac{11}{16}$	$1\frac{1}{4}$	$1\frac{1}{4}$	$\frac{7}{8}$
$1\frac{1}{16}--->1\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$5\frac{3}{4}--->6\frac{15}{16}$	$1\frac{1}{2}$	$1\frac{1}{2}$	1
$1\frac{13}{16}--->2\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	$7--->9\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{3}{4}$	1
$2\frac{1}{16}-->2\frac{13}{16}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{7}{16}$	$10-->12\frac{1}{2}$	2	2	1
$2\frac{7}{8}--->3\frac{5}{16}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$				

Table (5.2) Mechanical Properties of Metals (Inch system)

Material	Ultimate tensile strength (kpsi)	Elastic limit			Young's Modulus (kpsi)	Modulus of rigidity (kpsi)
		Tensile (kpsi)	Compressive (kpsi)	Shear (kpsi)		
Steel casting SAE 0022	60	25	33	15	29000	11200
Steel casting SAE 0030	72	30	39	17	29000	11200
Steel casting SAE 0050	80	32	43	20	29000	11600
Alloy steel casting SAE 090, ASTM A-142	90	60	60	36	29000	11300
Stainless steel: C 0.10, Cr 12, Ni 1	190	130	130	80	29000	11200
Stainless steel: C 0.10, Mn 0.4, Si 0.35, Cr 12, Ni 0.6	105	60	60	36	29000	11200
Stainless steel: SAE 30905	96	48	48	30	30000	12000
Carbon steel SAE1010	54	31	31	20	30000	11700
Carbon steel SAE1020	62	35	35	22	30300	11600
Carbon steel SAE1030	75	42	42	26	30200	11500
Carbon steel SAE1040	90	50	50	30	30000	11400
Carbon steel SAE1050	95	52	52	35	29800	11400
Carbon steel SAE1095	120	60	60	36	29700	11400
Carbon steel SAE1120	62	34	34	22	30200	11600
Nickel steel SAE2320	70	45	45	27	29700	12000
Nickel steel SAE2340	120	95	100	60	30000	12100
Cr-Ni steel SAE 3140	155	95	100	57	30500	12500
Cr-Ni steel SAE 3240	160	120	140	72	30500	12500
Cr-V steel SAE 6150	200	170	190	100	31000	13000
Cr-Ni-V steel	160	130	130	80	30500	12500
Nitraalloy Steel	125	90	120	55	29000	11600
Wrought iron	47	26	24	16	27000	10000

Table (5.3) Mechanical Properties of Metals (Metric system)

Material	Ultimate tensile strength[1]	Elastic limit (MN/m^2)			Young's Modulus (GN/m^2)	Modulus of rigidity (GN/m^2)
		Tensile	Compressive	Shear		
Steel casting SAE 0022	410	175	230	115	200	76
Steel casting SAE 0030	500	210	270	120	200	76
Steel casting SAE 0050	550	220	300	140	200	79
Alloy steel casting SAE 090, ASTM A-142	620	420	420	250	200	77
Stainless steel: C 0.10, Cr 12, Ni 1	1250	900	900	550	200	76
Stainless steel: C 0.10, Mn 0.4, Si 0.35, Cr 12, Ni 0.6	720	420	420	250	200	76
Stainless steel: SAE 30905	660	330	330	210	207	83
Carbon steel SAE1010	375	215	215	140	207	83
Carbon steel SAE1020	430	240	240	150	228	80
Carbon steel SAE1030	520	290	290	180	220	79
Carbon steel SAE1040	620	345	345	210	207	78
Carbon steel SAE1050	655	360	360	250	205	77
Carbon steel SAE1095	850	425	425	345	204	77
Carbon steel SAE1120	430	255	255	150	204	77
Nickel steel SAE2320	480	310	310	190	204	83
Nickel steel SAE2340	850	655	700	415	207	83
Cr-Ni steel SAE 3140	1,100	660	700	395	210	86
Cr-Ni steel SAE 3240	1,100	830	960	500	210	86
Cr-V steel SAE 6150	1,400	1,200	1,300	700	214	90
Cr-Ni-V steel	1,100	900	900	550	210	86
Nitraalloy Steel	850	620	820	380	200	79
Wrought iron	320	180	165	110	186	70

Example:

Find suitable dimensions of a square key to fit into $3\frac{7}{16}$ inch diameter shaft. The shaft transmits 95 hp at a speed of 200 rpm. The key is made of steel SAE 1010. Take safety factor of 2.5 and stress concentration factor $k' = 1.6$

Solution:

Exercises:

1. A shaft transmits a torque of 150 Nm to a pulley through a square key. The key is made of steel SAE 1020. Taking a safety factor of 2 determine suitable dimension for the key if the diameter of the shaft is 50 mm.

2. A power of 10 kW is transmitted from a 40 mm diameter shaft to a spur gear through a square key. The shaft rotates at 1000 rpm. Select a suitable material for the key and determine its dimensions. Take factor of safety = 1.5

3. A shaft transmits a torque of 100 lbin. to a pulley through a square key. The. Taking a safety factor of 2 select a suitable material and determine dimension for the key if the diameter of the shaft is 2 in .