

Load Stress and Failure

Safety factor

A component subjected to a solitary load will be considered in the first instance. This load is interpreted in the context of the component's nature and duty - thus **load** usually implies a transverse force in the case of a beam component, or a longitudinal compressive force in a column, or a torque in the case of a shaft, or a pressure in a fluid containment vessel, and so on.

There are two completely different expressions of the load, which have important consequences for the component :

- the extrinsic **actual load** is the load exerted on the component by its surrounds, and
- the intrinsic **maximum load** is the largest load that the component can withstand without failure; the maximum load is a property of the component, a function of its dimensions and material.

There are many reasons why the actual load on a particular component at a particular time will differ from the supposed value, including -

- The inherent variability of the load (eg. in practice the mass of a "ten tonne truck" will depend on the load it's carrying),
- Static indeterminacy (when components share the load in proportion to their elastic responses),
- Dynamic (or *shock*) effects - for example if a weight **W** is dropped from a height **h** onto an elastic component of stiffness **k**, see **fig.(2.1)**, then the peak force in the component is

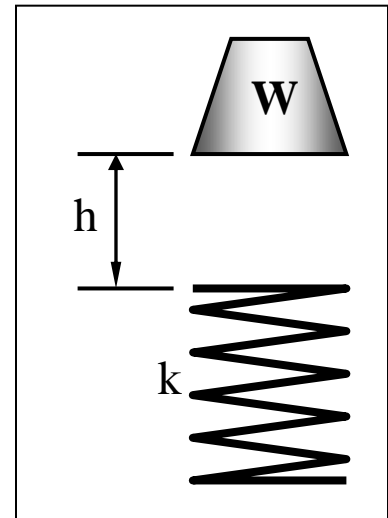


Fig. (2.1)

$$F_{\text{dyn}} = \text{dynamic magnification factor (dmf)} * W \quad (2.1)$$

Elementary energy methods give $dmf = [1 + (1 + 2hk/W)^{1/2}]$ for this type of loading, demonstrating that releasing a load W suddenly, *even from zero height*, induces a peak force of at least twice that arising from gradual release - *the effective actual load is at least twice its nominal or supposed value*.

Similarly the maximum load which a particular component can withstand at a particular moment in time will differ from the nominal maximum load due to many factors including :-

- **dimensions** differing from their nominal or expected values - eg. we might look at component diameter, calling it 50 mm and figuring the nominal maximum load on that basis; but the diameter may in fact be 47 mm, or 54 mm, or more likely vary along the branch;
- **material strength** differing from its nominal value due in turn to variations in material composition or heat treatment, to unsuspected flaws (is there an undetected split or other irregularity in our component branch?).

Deviations may be highly significant or they may be insignificant - it's important that we have an idea of their magnitudes. Sometimes we haven't much of a clue and therefore have to fear the worst and make suitable allowance for our ignorance.

Clearly a component is safe only if the actual load applied to the component does not exceed the component's inherent maximum sustainable load. The degree of safety is usually expressed by the safety factor, n :-

$$\begin{aligned} n &= \text{maximum allowable load} / \text{actual load} \\ (2.2) \\ &= F_{\text{max}} / F \end{aligned}$$

and it follows that :

if $n = 1$ then the component is on the point of failure

if $n < 1$ then the component is in a failed state

if $n > 1$ then the component is safe.

The safety factor is usually expressed as a ratio of nominal loads. A higher value of the safety factor seems to indicate a safer component - however this is not necessarily the case as the inevitable variations must be kept in mind.

In fig. (2.2A), $n = 1.25$ based on nominal values, but because of the relatively large variations in both actual and maximum loads, there is a significant probability of the actual load exceeding the maximum, and hence of failure.

For a negligible failure probability with these levels of variations, the nominal safety factor must be increased by eg. reducing the actual load applied to the component, as indicated in Fig.(2.2B) - alternatively the maximum load could be increased, by increasing the material's strength or the component's dimensions.

Fig.(2.2C) illustrates how the probability of failure may be decreased also by reducing the variations of the actual and/or maximum loads. This can be accomplished by a better understanding of what's going on - we'll amplify this later.

When an assemblage of components is subjected to a single load, the assembly's safety factor is the smallest of the component safety factors - 'a chain is only as strong as its weakest link'. If a component or an assemblage is subjected to a number of different simultaneous loads, then the concept of a single safety factor may be inappropriate - but nevertheless all potential failure mechanisms must be investigated when deciding whether an implement is safe to use or not.

IF, *and only If*, the stress in a component is proportional to the actual load on the component, then the safety factor may be interpreted

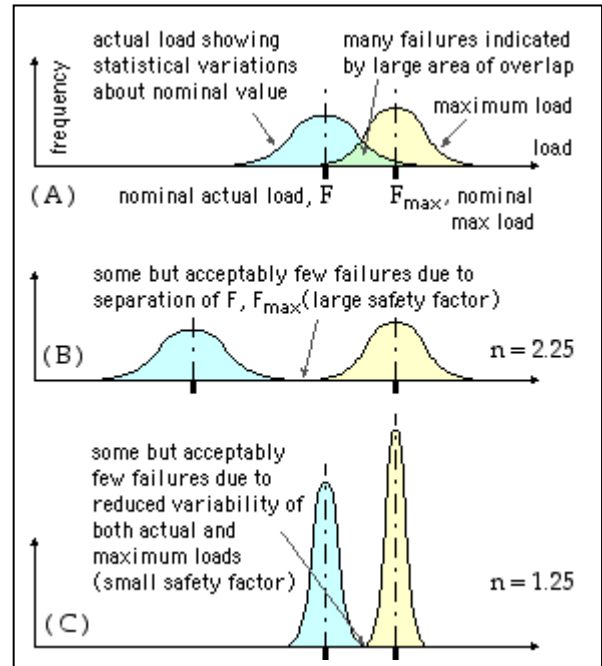


Fig. (2.2) variation of actual and maximum allowable loads

also as a stress ratio :- $n = S / s$

(2.3)

where :

S is the strength of the component's material,

s is the stress in the component due to the actual load, F

Such an interpretation is evidently inapplicable to assemblages. Stresses are proportional to load for the majority of practical elements, so we usually assume that the stress ratio interpretation (2.3) applies. But this is NOT the case with some not uncommon failure mechanisms (examined later) such as buckling and Hertzian contact - in these cases we must fall back on the fundamental load interpretation (2.2).

The strength of a material is the maximum stress it can withstand without failure; it is obtained from a tensile test on a specimen of the material. Common metals follow stress-strain relations of the forms illustrated below, see fig. (2.3):

The stress in a brittle material, Fig.(2.3A), cannot exceed the ultimate strength, S_u . Ductile materials also display such an upper limit,

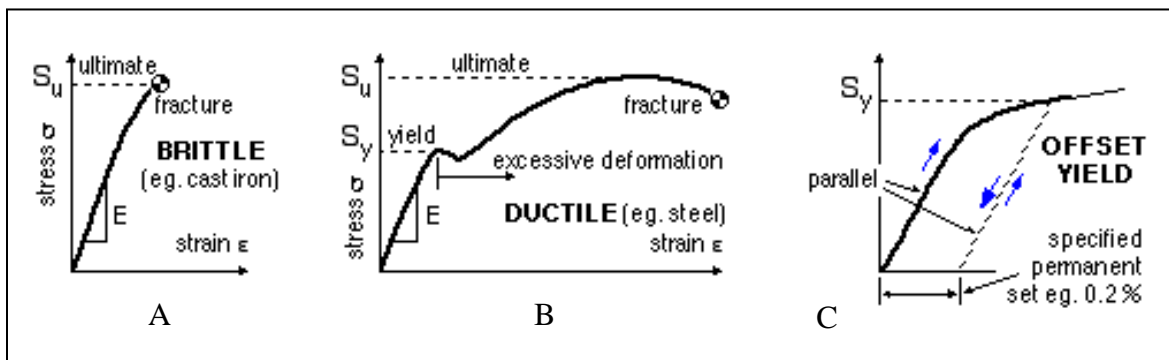


Fig. (2.3) stress strain relations

Fig.(2.3B), but the yield strength, S_y , is often more relevant as it forms a bound above which plasticity and excessive deformation may occur. Most modern ductile materials do not possess a distinct yield so in this case an artificial value is usually defined- *the offset yield*, Fig.(2.3C)- based upon some maximum acceptable permanent deformation (eg 0.2%) remaining after load release. Yields and ultimates are material properties; representative values for many materials are cited in the literature.

Let us derive a design equation for the simplest of all components - the tensile bar. The bar's cross-sectional area is A , it is subjected to a tensile force P and the strength of its material is S - which may be the ultimate if fracture is important, or the yield if the material is ductile and excessive deformation is relevant.

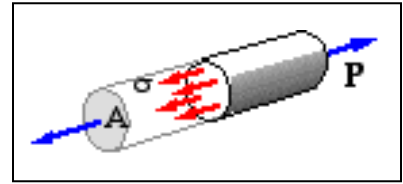


Fig. (2.4)

Assuming uniform stress across the cross-section, the maximum load that the component can sustain occurs when the stress s reaches the material strength, and is $P_{\max} = A.S$ - this expression when combined with (2.1) leads to the design equation for direct normal stress, namely :-

$$n P = A S \quad (2.4)$$

The design equation embodies conveniently and directly the four aspects mentioned previously - *safety, load, dimensions and material* (strength). The equation may be used either :

- for analysis to determine the degree of safety $n = A S / P$ for given S , A , P , or
- for synthesis (design) to ascertain the dimensions required $A \geq n P / S$ to withstand a given P with material of strength S and specified degree of safety n .

Codes often cite a material's allowable or *design stress*, $s_d = S/n$ as a characteristic in its own right, rather than specify both the strength and the necessary safety factor. The required area would then be $A \geq P/s_d$.

Equation (2.4) applies also to compressive loading, however components in compression are usually more likely to fail by buckling than by any strength limitation being exceeded. Buckling (ie. geometric instability) is dictated as much by the component's overall geometry as by the inherent strength of the component's material. A cardboard tube can withstand a larger compressive load than a steel wire.

Suitable nominal factors of safety for use in elementary design work are as follows :

Table (2.1) Suggested Safety (design) Factors for Elementary Work

1	1.25 - 1.5	for exceptionally reliable materials used under controllable conditions and subjected to loads and stresses that can be determined with certainty - used almost invariably where low weight is a particularly important consideration
2	1.5 - 2	for well-known materials under reasonably constant environmental conditions, subjected to loads and stresses that can be determined readily.
3	2 - 2.5	for average materials operated in ordinary environments and subjected to loads and stresses that can be determined.
4	2.5 - 3	for less tried materials or for brittle materials under average conditions of environment, load and stress.
5	3 - 4	for untried materials used under average conditions of environment, load and stress.
6	3 - 4	should also be used with better-known materials that are to be used in uncertain environments or subject to uncertain stresses.
7		Repeated loads : the factors established in items 1 to 6 are acceptable but must be applied to the endurance limit (ie. a fatigue strength) rather than to the yield strength of the material.
8		Impact forces : the factors given in items 3 to 6 are acceptable, but an impact factor (the above <i>dynamic magnification factor</i>) should be included.
9		Brittle materials : where the ultimate strength is used as the theoretical maximum, the factors presented in items 1 to 6 should be approximately doubled.
10		Where higher factors might appear desirable, a more thorough analysis of the problem should be undertaken before deciding on their use.

The table illustrates clearly the need to increase safety factors when designing with loads either actual or maximum which are not known with certainty. In order to avoid waste of material to cater for ignorance, we try to forecast loads as accurately as possible. For example, rather than use a large design factor to allow for unknown shock effects, we may

obtain a realistic dynamic magnification factor from the literature citing other folks' experience with similar components, and then increase the nominal actual load by this *dmf*. Statically determinate assemblies are generally preferred to indeterminate because component loads are predictable from simple statics and do not rely on a complex interaction between the components.

For the same reason we employ mathematical models which describe real behaviour as accurately as possible - rather than use simplistic models which are known to be poor predictors. Don't confuse a model's descriptive accuracy with its numerical accuracy - a mathematical model might give an answer to 53 decimal places, but it may describe a component's actual behaviour so simplistically that only one decimal place is significant.

Design factors are increased also when the consequences of failure - economic, social, environmental or political - are serious. For example, a local manufacturer of processing machinery intended for the headwaters of a remote River, doubled the size of every motor predicted by the design process because of the delay in providing replacements if failure should occur due to overload. This is another example of the designer having to foresee the whole future life of an artefact.

The tensile component is the simplest of all machine members. Investigation into the safety of a more complex component involves searching over all *elements* of the component (ie. over all locations in the component) to find the weakest link - the element with the smallest safety factor. In practice for a given component only a few elements are potentially critical, so an intelligent search is not necessarily lingering. At each element, safety is assessed by means of the following general approach :-

1. Find the *loads on the component*, given the loads on the machine or structure (assemblage). If the assemblage is statically determinate then free bodies will give the components' loads immediately, but if it is statically indeterminate then a three-branched attack based on *equilibrium*, *compatibility* and the

- constitutive laws* of the assemblage's individual members must be carried out.
2. Ascertain the stress components (often Cartesian) at the element in question again by the use of free bodies. This is essentially a superposition of *load building blocks* such as tension (or compression), shear, bending and torsion - though others will be encountered. If the member is itself indeterminate then the three-branched attack mentioned above must be applied to elements within the member - it will be recalled that the elementary equations for bending and torsional stresses were deduced in this manner based on an assumed deformation geometry.
 3. Resolution of the stress components into *principal stresses*, using either analytic techniques or Mohr's circles. We shall not consider tensor resolution of the general 3-dimensional state - it is somewhat complex and seldom encountered in routine design.
 4. Implementation of an appropriate *failure theory* whereby the biaxial or triaxial stresses at the element may be correlated with material strengths, which are derived from uniaxial tests, to evaluate the safety factor.

These four steps will be examined individually in the following sections, and will be exemplified in setting up the design equation for a very common component - a shaft of circular cross-section subjected to simultaneous bending and torsion.

But before embarking on this and writing down stress equations such as

$s = P/A$, we've got to appreciate the limitations of such mathematical models - we must be able to visualise qualitatively how stresses vary throughout a body and to appreciate the concept of stress concentration.

For example consider equation (2.4) applied to the assembly comprising two short coaxial bars, 1 and 2, of cross-sectional areas 100 & 200 sq.mm and strengths 600 & 400 MPa respectively (*recall that 1 MPa is*

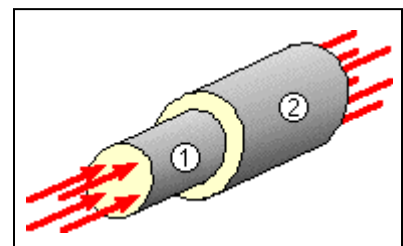


Fig.(2.5)

equivalent to 1 N/sq.mm).

The assembly is compressed by 50 kN distributed uniformly over each end. Is the assembly safe or unsafe ?

This example illustrates conclusively the need to visualise *qualitatively* how stresses vary throughout a body and to appreciate the concept of stress concentration - let's see how we go about this

Stress concentration

We shall use a tightened nut and bolt, in which the detailed stress variation is extremely complicated, to illustrate some of the simplifying assumptions which are usually made in routine analysis and design. The leftmost sketch in Fig.(2.6) shows an assembly comprising a nut and bolt which fasten two components together. The nut and bolt pair can be regarded as a sub-assembly or component in its own right. Before any identification of the stresses can be attempted, all external effects on this sub-assembly must be known. A free body of the nut and bolt demonstrates that the external load on them is the force P due to contact over annular areas with the two fastened components.

The flow analogy is useful when visualising how stress is transmitted through a loaded component. In the analogy, lines of force (or *force paths*) in the component are likened to streamlines in a fluid channel whose shape is similar to that of the component - fluid enters and leaves the channel at locations which correspond to the areas where the external loads are applied to the component.

A complicated streamline pattern infers a complex stress situation. Stresses are low where the streamlines are widely spaced. Stresses are high where the streamlines are bunched together due to geometric shape variations - the more sudden these variations, the higher the local stresses. This last is known as **stress concentration**. Geometric irregularities give rise to non-uniform stresses.

A free body of portion of the bolt which includes the bolt head shows that the external load P is equilibrated by stress s in the shank. If the free body boundary cuts the shank in the middle then the stress is **uniform** across the shank cross-section of area A - that is $s = P/A$.

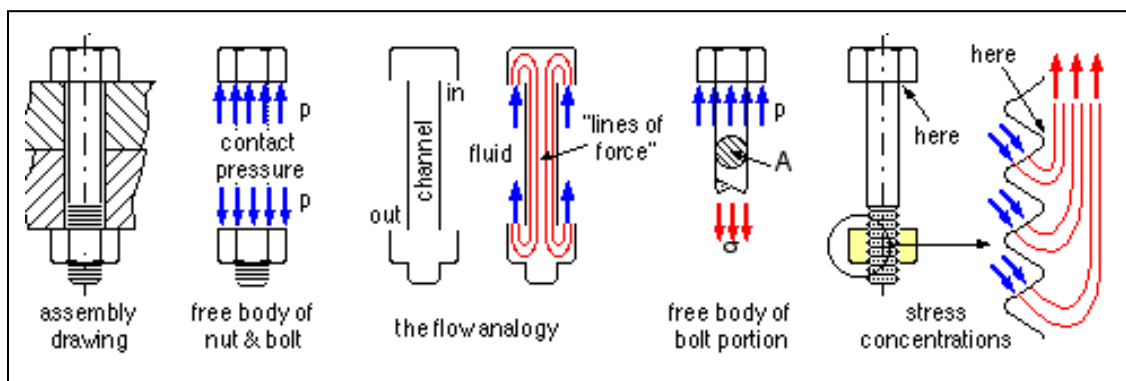


Fig. (2.6)

But if the boundary lies close to the head or to the thread and nut, then stress concentration occurs and the maximum stress is greater than P/A .

If it is required to calculate the elastic extension, d , of a bolt whose length L is relatively large, then a first approximation neglects stress concentration to obtain $d = Le = L(s/E) = LP/AE = P/k$ where the stiffness, k , of the bolt is defined as $k = AE/L$. If the emphasis is on bolt safety rather than on deflection, then the core area A_c should be used for stress calculations - this minimum area in the force transmission path is formed by a transverse plane cutting the bolt thread between the plain shank and the nut's contact face. The rightmost sketch above employs similar reasoning to illustrate details of the complex transfer of force across the thread into the bolt, together with the high stress concentration in way of the thread root, which may lead to localised yielding.

☉ Stress concentration factor:

The presence of a discontinuity increases the stress at and near the discontinuity. The maximum theoretical increase in stress compared with average or nominal stress is represented by the form factor K , (*can be obtained from diagrams for various discontinuities and loading*). The actual stress increase in machine parts is smaller than that given by the factor K ., because of the elasticity and plasticity of the metals.

The actual weakening effect is called the **stress concentration factor (K')**. The best way to find the stress concentration factor in a machine part is by testing the part under actual or simulated service condition. A simpler although less accurate, method can be obtained from the expression:

$$K' = 1 + q(K - 1)$$

Where q is called index of sensitivity of the material

Static loads:

Material	Index of sensitivity
Ductile material	0
Brittle material, <i>hardened steel</i>	0.1
Very brittle material, <i>quenched steel</i>	0.2
Cast iron	0

Impact loading:

Material	Index of sensitivity
Ductile and very soft material	0.4
Ductile material	0.61
Brittle material, <i>hardened steel</i>	1
Cast iron	0.5

For repeated loads

Material	Index of sensitivity		
	Annealed or soft	Heat treated and drawn at 1200 ^o F	Heat treated and drawn at 900 ^o F
Armco iron, 0.02% C	0.15 0.20
Carbon steel			
0.10% C	0.05 – 0.10
0.20% C (also cast steel)	0.10
0.30% C	0.18	0.35	0.45
0.50% C	0.26	0.40	0.50
0.85% C	0.45	0.57
Spring steel, 0.56% C, 2.3 Si rolled	0.38
SAE 3140, 0.37 C, 0.6 Cr, 1.3 Ni	0.25	0.45
Cr-Ni steel 0.8 Cr, 3.5 Ni	0.25
Stainless steel, 0.3 C, 8.3 cr, 19.7 Ni	0.16
Cast iron	0 – 0.05
Copper, electrolite	0.07
Duraluminum	0.05 – 0.13

Failure theories

We have seen that failure of a tensile member occurs when the stress caused by the actual load reaches the stress limit, the strength, of the member's material. Correlation of the actual stress with the maximum stress (strength) is straightforward in this case because they are both uniaxial. But how can we correlate the triaxial stress state in a component - whose material strength(s) is measured in uniaxial tests - to assess failure tendency? Unfortunately there is at present no fundamental rationale for any such correlation.

We therefore suggest some *attribute* of the stress state as a descriptor of that state, an attribute such as the maximum stress or the specific energy, and then compare the values of this attribute for the given component triaxial state on the one hand and the uniaxial test state on the other. This claim is the *failure theory* based upon the particular attribute selected; it is a useful theory only if its predictions are confirmed by experiment.

Currently no universal attribute has been identified which enables prediction of failure of both ductiles and brittles to an acceptable degree of accuracy. To appreciate why this is so, it is useful to recall the differences in behaviour of ductiles and brittles, as outlined in fig. (2.7) below.

Under normal stress, ductiles' compressive characteristics are approximately the same as their tensile characteristics (fig.(2.7B,C) below) whereas brittles are significantly weaker in tension - eg. $S_{uc}/S_{ut} = 3.5$ to 4 for the cast irons (S_{uc} and S_{ut} are the compressive & tensile ultimates).

A ductile's shear strength is about half its tensile strength whereas shear and tensile strengths are about the same in the case of a brittle.

Superposition of Mohr's circles of sizes corresponding to these proportions (fig.(2.7D) suggests that a shear-based attribute is relevant to ductiles, whereas a normal-based attribute to brittles. (*Note: the two superimposed circles correspond to completely different stress states - they have been superimposed only to highlight the circles' relative sizes.*)

This conclusion is reinforced when we consider the directions of the fracture surfaces under normal and shear loadings (fig(2.7E)). Thus in tension, ductiles fail at 45° to the load sense, which is the direction of maximum shear. This same 45° fracture sense is found with brittles under torsion, however in this case the Mohr's circle is centred on the

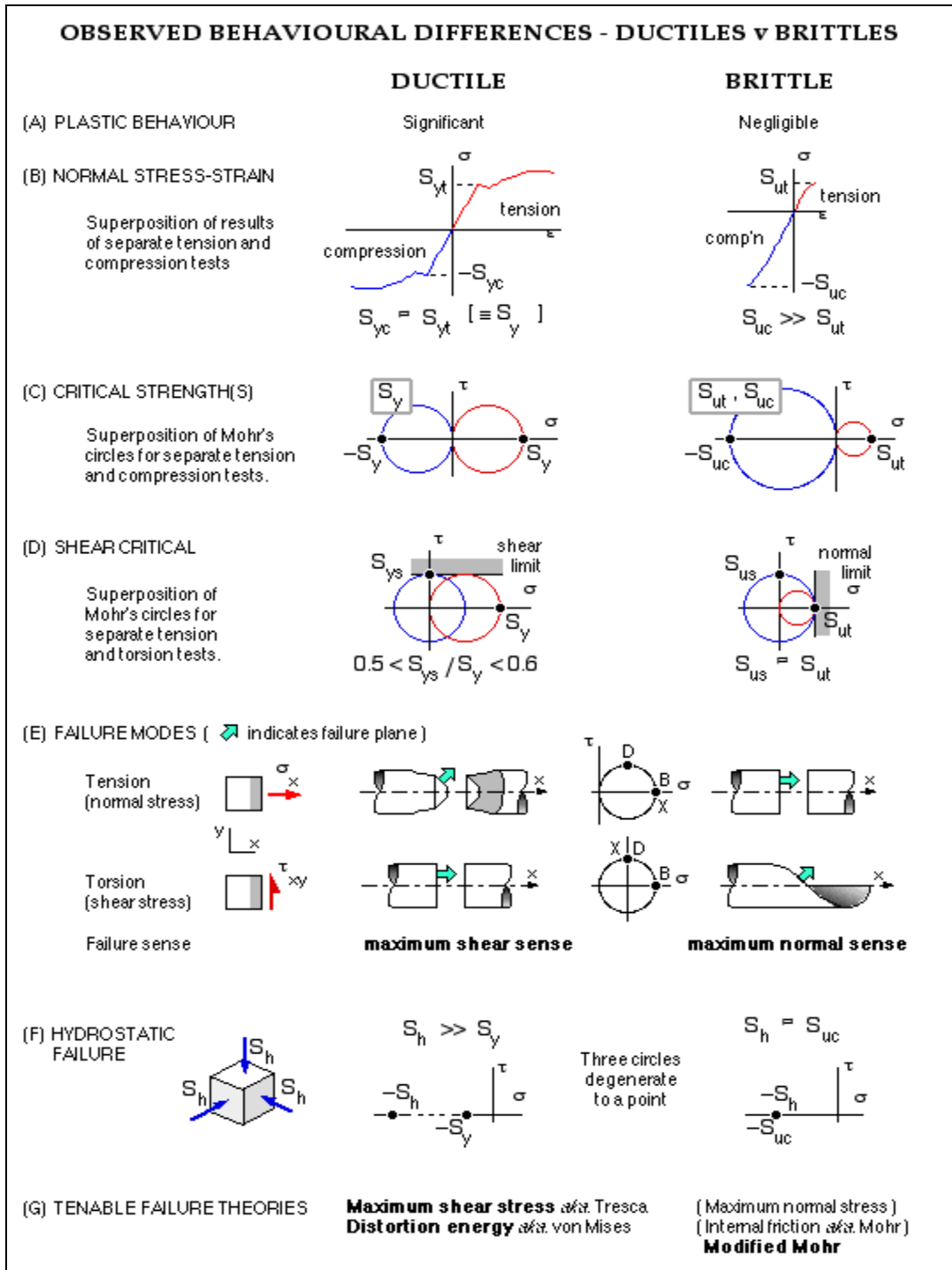


Fig. (2.7)

origin thus indicating a maximum normal attribute for brittles.

A final qualitative argument involves strength under hydrostatic pressure, for which the three Mohr's circles degenerate to a single point (fig.(2.7F). Brittles fail when the pressure corresponds approximately to the compressive strength, but it is difficult to assess when ductiles fail as the hydrostatic strength is so large - the absence of failure may be equated to the absence of shear.

The result of these observations is the adoption of the failure theories indicated - using

- shear-based attributes for ductiles
- normal-based attributes for brittles

These theories are examined below.

We refer to the *equivalent stress* S_e for a ductile material only, as the uniaxial stress which has the same failure tendency as a given triaxial state - on the basis of whichever attribute is relevant. The safety factor then follows from (2.4) as $n = S/S_e$.

1. *Distortion Energy Theory (also known as. Von Mises Theory):*

It may be shown that the specific distortion (or *shear strain*) energy for a linear material under the triaxial state (s_1, s_2, s_3) is proportional to:

$$[(S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_3 - S_1)^2].$$

For a uniaxial stress $(s_e, 0, 0)$, or merely s_e , this expression becomes $2s_e^2$. So s_e is equivalent to the triaxial system - ie. gives rise to the same distortion energy - if :-

$$2S_y = (S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_3 - S_1)^2 \quad (2.5)$$

2. *Maximum Shear Stress Theory (also known as. Tresca Theory):*

This theory postulates that if a uniaxial stress S_y , and a triaxial stress state give rise to the same maximum shear stress, then the failure tendencies of the two states are identical; that is, S_e is equivalent to the triaxial state. So, for the same diameter of the largest Mohr's circle :-

$$S_y = S_1 - S_2 \quad (2.6)$$

See fig.(2.8)

The intermediate principal, which happens to be S_2 in the sketch, has no effect upon failure tendency according to this theory.

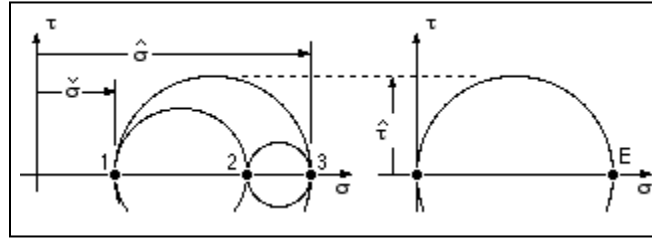


Fig. (2.8)

Either of the foregoing theories may be applied to ductiles: distortion energy represents experimental results somewhat better than does maximum shear stress. The maximum shear stress theory is always conservative. Both theories have their drawbacks in use - the distortion energy theory introduces non-linearities into otherwise linear problems, whereas the maximum shear stress theory requires the ordering of the principals. It is the best of the five.

3. **Maximum Stress theory (Rankine):**

$$S_1 = S_y$$

Hold well for brittle materials

4. **Maximum Shear Stress Theory (Guest - Tresca):**

$$\frac{1}{2}(S_1 - S_2) = \frac{1}{2} S_y$$

$$S_y = S_1 - S_2$$

Holds well for ductile materials

5. **Maximum Strain Energy per Volume Theory (Haigh):**

Failure occurs when the total strain energy in the complex stress system is equal to that at the yield point of the tensile stress.

$$\frac{1}{2E} [S_1^2 + S_2^2 + S_3^2 - 2\nu(S_1S_2 + S_2S_3 + S_3S_1)] = \frac{S_y^2}{2E}$$

$$S_1^2 + S_2^2 + S_3^2 - 2\nu(S_1S_2 + S_2S_3 + S_3S_1) = S_y^2$$

Good results for ductile materials

5. Maximum principal Strain Theory (Saint venant)

- Failure will occur when the maximum strain in the complex system equals that at yield point in the tensile test

$$\frac{S_1}{E} - \nu \frac{S_2}{E} - \nu \frac{S_3}{E} = \frac{S_y}{E}$$

$$S_1 - \nu S_2 - \nu S_3 = S_y$$

- **Holds well for cast iron**